

FIJI MATHEMATICS ASSOCIATION



FIJI MATHEMATICS COMPETITION (FMC) YEAR 13

Wednesday 6th September 2017

Time Allowed: 1 Hour 15 minutes

Note:

Calculators are NOT permitted.

Diagrams are NOT drawn to scale.

Instructions:

1. Print your **Name** in the space provided and Shade the circle corresponding to your **Year** on the answer sheet.
2. Shade the circle corresponding to your answer with dark pencil on the answer sheet provided.
3. Multiple answers **will not be** accepted.

Year 13

1. Evaluate $300 \times 8 \times 0.1 \times 0.001$

- A. 0.0024 B. 2.4 C. 0.024 D. 0.24 E. 24

2. $\frac{3}{16}$ when expressed as a decimal is:

- A. 0.375 B. 0.1875 C. 0.125 D. 0.25 E. 0.24

3. $(\sqrt{2} \sqrt{3})^6$ can be simplified as:

- A $\frac{1}{3}$ B. 1296 C. $6\sqrt{6}$ D. 36 E. 216

4. Which of the following trigonometric ratios is equal to $\sqrt{2}$?

- A. $\frac{\sin 30^\circ}{\sin 45^\circ}$ B. $\frac{\sin 45^\circ}{\sin 30^\circ}$ C. $\frac{\cos 30^\circ}{\cos 45^\circ}$ D. $\sin 45^\circ$ E. $\sin 30^\circ$

5. What is the horizontal asymptote of the rational function $f(x) = \frac{1-x^2}{(x+2)(x-3)}$

- A $y = -\frac{1}{6}$ B. $y = \frac{1}{6}$ C. $y = -1$ D. $y = 0$ E. $y = 1$

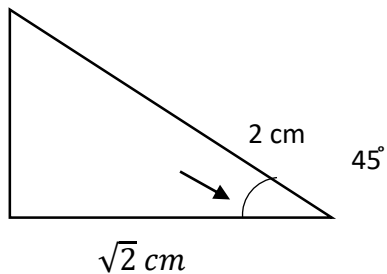
6. Simplify $\frac{100!}{98!}$

- A 99! B. 100! C. $\frac{50}{49}$ D. 990 E. 9900

7. The coefficient of x^{13} in the expansion of $(x^2 + \frac{2}{x})^8$ is:

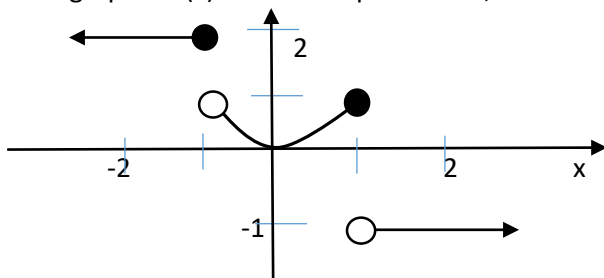
- A 256 B. 128 C. 64 D. 32 E. 16

8. What is the area (in cm^2) of the triangle given below?



- A $\frac{1}{2}$ B. 1 C. $\sqrt{2}$ D. 2 E. 4

Consider the graph of $f(x)$ to answer questions 9, 10 and 11.



Year 13

9. Find $\lim_{x \rightarrow -1} f(x)$

- A. -1 B. 1 C. 2 D. 0 E. Does not exist

10. Evaluate $f(-1)$

- A. -1 B. 1 C. 2 D. 0 E. Does not exist

11. The value of $f'(0.5)$ is

- A. 0 B. 1 C. > 0 D. < 0 E. Undefined

12. $\frac{2}{5}$ of students in a class are girls. If there are 8 more boys than girls how many students are there in the class?

- A. 16 B. 24 C. 20 D. 36 E. 40

13. The argument of complex number, $Z = \frac{-i}{2}$ is

- A. $-\pi/2$ B. $-\pi$ C. $-\pi/4$ D. $\pi/4$ E. π

14. If $x^2 + \frac{1}{x^2} = 27$, the value of $x - \frac{1}{x}$ is

- A. 4 B. 5 C. $\sqrt{27}$ D. 24 E. 25

15. Solve $\log_{\frac{1}{3}}(\frac{1}{243}) = 2x$

- A. $\frac{5}{2}$ B. -3 C. -4 D. -5 E. $-\frac{3}{2}$

16. If a and b are positive numbers such that $a^b = b^a$ and $b = 9a$ then the value of a is

- A. 9 B. $\frac{1}{9}$ C. $\sqrt[3]{9}$ D. $\sqrt[9]{9}$ E. $\sqrt[4]{3}$

Consider the two functions to answer questions 17 and 18

$$f(x) = x^2 + 2 \quad \text{and} \quad g(x) = \sqrt{x-3}$$

17. The expression $\text{fog}(x)$ can be simplified as:

- A. $x^2 + \sqrt{x-3}$ B. $\sqrt{x^2-1}$ C. $\sqrt{x^2+1}$ D. $x+1$ E. $x-1$

18. The domain of $\text{fog}(x)$ is:

- A. $x \geq -2$ B. $x \geq 3$ C. $x \leq -2$ D. $x \leq 3$ E. $x \in \mathbb{R}$

19. The volume of a balloon is given as $V = \frac{2}{3} \pi r^3$, where V is volume and r is radius. If air is pumped into the balloon, the rate of change of the balloon's volume is given by;

- A. $4\pi r^2$ B. $4\pi r$ C. $2\pi r^2$ D. $2\pi r$ E. $4\pi r^4$

Year 13

20. Given $\sin \theta = \frac{4}{5}$, find the value of $\sin 2\theta$. ($\sin 2\theta = 2 \sin \theta \cos \theta$)

- A. 2 B. 4 C. $\frac{12}{25}$ D. $\frac{16}{25}$ E. $\frac{24}{25}$

21. The exact value of $\frac{\sin^2 30^\circ \cos 30^\circ}{\sin^2 60^\circ \cos 60^\circ}$ is:

- A. 1 B. 2 C. $\frac{\sqrt{3}}{2}$ D. $\frac{3}{4}$ E. $\frac{1}{\sqrt{3}}$

22. The values of A and B in the expression $\frac{3-x}{(x+1)(x-1)} = \frac{A}{x+1} + \frac{B}{x-1}$ respectively are:

- A. 2 and 1 B. -2 and 1 C. -2 and -1 D. 2 and -1 E. 3 and -1

23. A couple has 3 children, what is the probability that at least 2 are girls?

- A. $\frac{1}{4}$ B. $\frac{1}{2}$ C. $\frac{3}{8}$ D. $\frac{7}{8}$ E. $\frac{1}{8}$

Consider the sequence a_n to answer questions 24 and 25; $a_n = \frac{2n^2+n}{n^3+2}$

24. Find a_3

- A. 0 B. 1 C. 2 D. $\frac{21}{29}$ E. $\frac{36}{66}$

25. Which of the following is the greatest lower bound of the sequence a_n

- A. 0 B. $\frac{1}{2}$ C. 1 D. 2 E. Does not exist

26. The product of 2 numbers is 1575 and their quotient is $\frac{9}{7}$. What is the sum of the numbers?

- A. 74 B. 78 C. 80 D. 90 E. 94

27. Evaluate $\int_{e^1}^{e^2} \frac{1}{x} \cdot dx$

- A. -1 B. 1 C. 2 D. $\ln 1$ E. $\ln 2$

28. A list of positive integers has a median of 8, a mode of 9 and a mean of 10. What is the smallest possible number of integers in the list?

- A. 5 B. 6 C. 7 D. 8 E. 9

29. if $y = 3 \sqrt[3]{\cos x}$ find $\frac{dy}{dx}$ ($\frac{d}{dx} (\cos x) = -\sin x$)

- A. $\sin x \sqrt[3]{\cos^2 x}$ B. $-\sin x \sqrt[3]{\cos^2 x}$ C. $\frac{\sin x}{\sqrt[3]{\cos^4 x}}$ D. $\frac{-\sin x}{\sqrt[3]{\cos^2 x}}$ E. $\frac{\sqrt[3]{\cos^4 x}}{\sin x}$

30.. If $x^2 - 3x = 2$ then $x^4 - 6x^3 + 9x^2 + 21$ equals

- A. 5 B. 25 C. 42 D. 67 E. 81